

were throughout their greater part of the usual fair weather type, the potential being neither specially high, specially low, nor specially variable. There were, however, two intervals, between 8.40 and 9.20 p.m. on May 19, and between 1.30 and 3 a.m. on May 20, when there were rapid oscillations and negative potentials, which were not accompanied—as is usually the case—by a rainfall visible in the rain-gauge curves. Thunderstorms were, however, in active progress at the time at no great distance, a good many peals of thunder being audible in Richmond; there was thus nothing in the electrical phenomena that is not adequately accounted for by the observed meteorological conditions.

May 21.

C. CHREE.

The Magic Square of Sixteen Cells. A New and Completely General Formula.

THE ancient problem: *To construct a Magic Square with sixteen consecutive integers*, may be regarded as a special case of the general problem: *To construct a Magic Square with any sixteen positive integers, no two of which shall be identical*. The solution of the problem thus generally enunciated throws much new light upon the ancient special one, and will, in fact, enable us to classify and tabulate its 880 known solutions (8×880 , if we admit reversals and reflections of the same square to be "different") much more scientifically than has hitherto been done.

The following is the completely general formula for the Magic of Sixteen Cells:—

$A-a$	$C+a+c$	$B+b-c$	$D-b$
$D+a-d$	B	C	$A-a+d$
$C-b+d$	A	D	$B+b-d$
$B+b$	$D-a-c$	$A-b+c$	$C+a$

For (1) this formula obviously represents a Magic Square, since every row, every column, and both the central diagonals sum to $A+B+C+D$.

Also (2) it is a function of eight independent variables.

Let S be the sum of our sixteen unknown quantities; then the constant total of the square will $=S/4$. If three of the rows sum to $S/4$, the fourth row must do the same; similarly with the columns.

Hence only eight of the ten given conditions are independent; we have to solve eight simultaneous linear equations involving sixteen unknown quantities. The solution, if general, must thus involve eight arbitrary constants. Therefore the above solution, which does involve eight arbitrary constants, is a perfectly general one.

I proceed to a numerical example. If $A=10$, $B=12$, $C=8$, $D=5$, $a=8$, $b=-9$, $c=-10$, $d=2$, our formula gives us a Magic summing in every direction to 35:—

2	6	13	14
11	12	8	4
19	10	5	1
3	7	9	16

It will be noticed that the number 19 is used, and the number 15 is not.

We have here an example of a Magic in its simplest form, with none of the superfluous (accidental) relations such as appear among the components when those numbers happen to be consecutive; and we see that the "complementary pairs" (each summing to half the constant total) upon which previous writers have laid such stress are a purely adventitious feature, and have no real connection with the laws of construction of the square.

In the fourth volume of the "Récréations Mathématiques" of Edouard Lucas (Paris, 1894) are set out three theorems and three corollaries, enunciating various equalities which must exist between the component numbers of every Magic of Sixteen Cells. The proof of these takes up four pages and a half, and requires twelve illustrative diagrams. My formula proves them all by simple inspection.

If, in the formula, $a=b$, the square assumes the type which Frénicle designated by the letter δ .¹ If $a=-b$, it assumes the type which Frénicle, in his table, left unmarked. Of the latter type, there are exactly 120 in consecutive numbers. I append an example of each type:—

δ			
1	12	13	8
16	9	4	5
2	7	14	11
15	6	3	10

1	9	16	8
7	15	10	2
14	4	5	11
12	6	3	13

($A=7$; $B=9$; $C=4$; $D=14$;
 $a=6$; $b=6$; $c=2$; $d=4$.)

($A=4$; $B=15$; $C=10$; $D=5$;
 $a=3$; $b=-3$; $c=-4$; $d=1$.)

It must be borne in mind, however, that a complete numerical solution of the δ type necessarily includes the squares which Frénicle marked α and β , because both of these are, algebraically, particular cases of the δ form.

My formula readily supplies an infinity of solutions of the problem, *To construct a Magic Square with sixteen different prime numbers*. The following example (first published by me in the *Pall Mall Gazette* of February 26 last) omits two only out of the first eighteen odd primes, and sums to a far smaller constant than any other investigator has been able to obtain:—

1	47	13	53
61	17	31	5
29	7	59	19
23	43	11	37

($A=7$; $B=17$; $C=31$; $D=59$;
 $a=6$; $b=6$; $c=10$; $d=4$.)

It is obvious that every 4^2 Magic formed by the addition of two Latin squares is divided into equal quarters. No proof, however, has up to now been given of the "converse" of this proposition. I will deduce the theorem from my general formula.

Theorem.—Every 4^2 Magic in equal quarters can be expressed as the sum of two Latin squares.

That the form of the result may be more convenient, I

¹ "Ouvrages de Mathématique." Par M. Frénicle. (La Haye, 1731.)

re-state my general formula, with interchanged letters, as below :—

$A+d$	$C-a-d$	$B+a-b$	$D+b$
$D+c-d$	B	C	$A-c+d$
$C+b-c$	A	D	$B-b+c$
$B-b$	$D+a+d$	$A-a+b$	$C-d$

The condition that each quarter shall be equal to $A+B+C+D$ is obviously $a-c+d=0$. Substituting $-a+c$ for d , the universal formula for a square in equal quarters is therefore :—

$A-a+c$	$C-c$	$B+a-b$	$D+b$
$D+a$	B	C	$A-a$
$C+b-c$	A	D	$B-b+c$
$B-b$	$D+c$	$A-a+b$	$C+a-c$

which, by putting $A+a$, $B+b$, $C+c$, for A , B , C , respectively, becomes :—

$A+c$	C	$B+a$	$D+b$
$D+a$	$B+b$	$C+c$	A
$C+b$	$A+a$	D	$B+c$
B	$D+c$	$A+b$	$C+a$

This is the familiar traditional form, being the addition of one Latin square (A , B , C , D) to another (a , b , c). It is usually written (inaccurately) as if it involved eight arbitrary variables, instead of seven.

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Magnetic Deflection of β Rays.

THE nature of the emission of α rays from radio-active bodies, and the mechanism of their absorption when passing through matter, are well known from the experiments of Rutherford, Bragg, and others.

As regards β rays, our knowledge is not so complete. Although in recent years a large number of experiments have been undertaken in order to study the laws of their absorption, there still remains considerable doubt concerning several fundamental points. From the study of the

absorption of β rays emitted from different radio-active substances, Otto Hahn and Lise Meitner arrived at the conclusion that the β rays, in the same way as α rays, are characterised by a definite initial velocity of expulsion. For different β -ray products the velocity may, of course, be different, but for a simple substance this velocity is characteristic of the rays. It was assumed by Hahn and Meitner that a homogeneous substance could be recognised as such by the exponential law of absorption by aluminium of the β rays which are emitted.

The experiments of W. Wilson were not in accord with this hypothesis. He found that the exponential law is not a measure of the homogeneity of the radiation, but, on the contrary, that homogeneous rays are absorbed according to a linear law.

In addition, the experiments of Kaufmann and Bucherer, who obtained a continuous magnetic spectrum of β rays in their determination of e/m and v for those rays, appeared to be contrary to the view of Hahn and Meitner. Such a spectrum could not be obtained on the assumption of groups of homogeneous β rays.

During the last few months the authors have investigated by a photographic method the magnetic deflection of β rays, and were able to show that in some cases very well-defined lines of deflection can be obtained. Experiments were especially successful when the active deposit from thorium served as source of radiation. As Hahn and Meitner have shown, this contains two groups of β rays (ThA and ThD). The authors obtained in this case two distinctly separated lines in the magnetic field. The line due to thorium A , which was further deflected, was nearly as well defined as if it were produced by α rays. Of course, by use of a stronger field, a third line, fairly well marked, was absorbed very near the ThA line, the source of which we are not yet quite certain.

But it is of interest that Hahn and Meitner recently discovered a new easily absorbed β radiation in ThX , and that the photographic impression, when using thorium X , really gave one more line as when using the active deposit alone.

Mesothorium gave a number of well-separated lines (about five or six). In this case the absorption experiments of Hahn and Meitner had already indicated a complex β radiation.

In the case of radium we have not, so far, been able to obtain single bands. This may perhaps be ascribed to the fact that the β rays from the radium products do not differ much in their velocities, and that the bands were consequently superposed, the intensity of the magnetic field being only about 80 Gauss. As a whole, the photographic impressions produced by the hard β rays are not very clear, since the rays pass through the photographic film without appreciable absorption, giving rise to a secondary radiation which fogs the plate.

The authors have proved by their experiments, at least for several of the radio-active elements, that these elements emit groups of β rays of definite velocity for which e/m and v can be separately determined.

A more detailed account of these experiments will be published elsewhere.

OTTO VON BAEYER.

OTTO HAHN.

Berlin, May 1.

Peripatus papuensis.

At the end of last June I received from Mr. A. E. Pratt, the well-known naturalist, a number of fine specimens of *Peripatus* which he and his son, Mr. F. B. Pratt, had found in New Guinea on their recent expedition to that island. This is the first time *Peripatus* has been found in New Guinea. It was found by Dr. Willey in New Britain in 1897, and by Mr. Muir and Mr. Kershaw in Ceram last year (see *NATURE*, July 1, 1909, p. 17, and *Quarterly Journal of Microscopical Science*, liii., 1909, p. 737). The New Guinea specimens were found in January, February, and March at Sarayu, at an elevation of 3500 feet in the Central Arfak Mountains. Mr. Pratt, in describing his discovery, writes as follows:—"After my son found the first specimen amongst the roots of the grass, we at once showed it to the natives, offering them a large knife (which is most valuable to them) for every specimen. Quite sixty of the natives were searching for